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AN INTRODUCTION TO THE MODELING AND ANALYSIS OF FLEXIBLE MANUFACTURING SYSTEMS WITH PETRI NETS

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AN INTRODUCTION TO THE MODELING AND ANALYSIS OF FLEXIBLE MANUFACTURING SYSTEMS WITH PETRI NETS

INTRODUCTION À LA MODELISATION ET À L'ANALYSE DES SYSTÈMES DE FABRICATION SOUPLES AU MOYEN DE RÉSEAUX DE PETRI

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ABSTRACT

Petri nets provide useful concepts and tools for studying the structure and control of concurrent systems. Petri nets have been used in the modeling of a wide variety of systems from computers to sociotechnical systems. This report discusses the use of Petri nets in the modeling and analysis of flexible manufacturing systems (FMS). Net concepts relevant to the manufacturing systems are introduced and qualitative properties such as the existence or absence of deadlock, safeness, and boundedness are discussed using the method of invariants. The modeling and analysis techniques are illustrated by means of simple manufacturing systems. The concepts of coloured Petri nets and their usefulness in the modeling of FMS is also discussed.

RÉSUMÉ

Les réseaux de Pétri constituent un concept et un outil utiles pour étudier la structure et la régulation de systèmes concourants. Ils ont été utilisés dans la modélisation d'une vaste gamme de systèmes allant des ordinateurs aux systèmes socio-techniques. Le présent rapport traite de l'utilisation de réseaux de Pétri dans la modélisation et l'analyse des systèmes de fabrication souples (SFS). Des concepts de réseaux pertinents aux systèmes de fabrication sont présentés, et des propriétés qualitatives telles que l'existence ou l'absence d'impasses, la sûreté et les limites sont analysées par la méthode des invariants. Les techniques de modélisation et d'analyse sont illustrées à l'aide de systèmes de fabrication simples. Les principes des réseaux de Pétri en couleur et leur utilité dans la modélisation des SFS sont aussi abordés.

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LIST OF SYMBOLS

Symbol

B	buffer place
B_p	matrix formed by p-invariants
C	incidence Matrix
C_{ij}	element in the ith row and jth column of C
C(p)	set of colours of place p
C(t)	set of colours of transition t
C^*	pseudo inverse of C
E_j	m-vector whose entries are zero except the jth entry
F	firing count vector
FMS	flexible manufacturing system(s)
G	Petri net
I_n	n × n unit matrix
I(p,t)	input function from place p to transition t
J_j	part type j
K(p)	Capacity of place p
M	marking of Petri net
M_o	initial marking
M(p)	marking at place p
O(p,t)	output function from transition t to palce p
P	set of places p; place P
p_i	place i
R(M)	set of all reachable markings from M
S_i	work station i
S_{ij}	processing of part type j on work station i
S_{ikj}	work station s_i processing kth operation on part type J_j
t_i	transition i
T	set of transitions; transpose
v_k	firing or control vector at the kth firing

CONTENTS (Cont'd)

W_{ji}	part type j waiting for work station i
W_{jk}	part type j waiting for kth operation
x	p-vector
x_i	ith component of x
y	t-vector
y_i	ith component of y
z	arbitrary m-vector

GREEK AND OTHER SYMBOLS

Symbol	
Δ	change
ε	is an element of
ϕ	empty or null set
$\|\cdot\|$	cardinality
$\|\|\cdot\|\|$	support of p or t-invariant
$\neq$	not equal to
$\cup$	union
$\cap$	intersection
$\times$	Cartesian product

AN INTRODUCTION TO THE MODELING AND ANALYSIS OF FLEXIBLE MANUFACTURING SYSTEMS WITH PETRI NETS

1.0 INTRODUCTION

A flexible manufacturing system (FMS) is a fully automated production system that can simultaneously process medium-sized volumes of a variety of part types. A generic FMS consists of a set of machines or work stations, linked by a material handling system and all under a central computer. There is also a provision for storage of work-in-process inventories, locally at the work stations and/or centrally at the system level. The tools are usually kept at each machine, but in some systems tools may be delivered to the machines from a central store by a special delivery system. In most machining systems jobs are mounted on pallets or fixtures which are limited in number. This introduces a constraint on the number of jobs that can be in the FMS at any time.

Several different types of flexibility can be present in an FMS. Browne et al. (1984) have described eight types of flexibilities. Machines have some degree of flexibility and do not require significant set-up time between successive jobs. During operations, the system can respond flexibily to unforeseen events, such as machine breakdown and temporary overloads, by rerouting parts to alleviate potential bottlenecks. These and other levels of flexibility allow an FMS to produce a different set of part types quickly, economically, in several different ways and at different production levels.

Flexible manufacturing systems are dynamic in nature as they evolve in time. However, unlike the traditional dynamical systems that model the behavior of physical devices and phenomena, they cannot, in general, be easily modeled by the system of ordinary or partial differential equations. Such systems are driven by the occurrence of discrete events, such as the start or completion of a task. These events trigger other events in the system according to some rules of operation. We call these systems discrete event dynamic systems.

To study the numerous complex interactions occurring in a manufacturing system, many modeling approaches have been proposed. Buzacott et al. (1986) and Kalkunte et al. (1986) have reviewed the modeling approaches based on queueing theory, mathematical programming, and perturbation analysis. Dubois et al. (1983), however, suggest the use of Petri nets and coloured Petri nets to model and analyze these systems. Besides these analytical methods, the simulation modeling technique is also used.

Petri nets (originated by Petri in his Ph.D. dissertation of 1962) have developed over the years into a useful tool for the description and analysis of concurrent systems (Peterson 1981). A Petri net can be represented by a bipartite directed graph. The graphical representation provides a user-friendly notation and a quick structural overview of the system. The Petri net theory deals with distributed systems and processes, and as such is well suited to the modeling of an FMS. It provides a rich mathematical framework to analyze the modeled system. Qualitative properties, such as absence of

deadlocks and finiteness of buffers and resources, can be easily investigated using the theory of invariants (Narahari et al. 1985a; Viswanadham et al. 1987). Besides these qualitative properties, Petri nets can also be used for the design of simulation and performance analyses by associating time to the various activities of an FMS (Alla et al. 1986; Balbo et al. 1987; Bruno et al. 1987; Pagnoni 1987).

The purpose of this report is to present the essential ideas that underlie the Petri net approach in the modeling and analysis of an FMS. Assuming no acquaintance with Petri nets we review in detail the relevant theory in Section 2. The fundamental concepts of concurrency and conflict (choice) are defined and explained. In Section 3, we discuss a procedure for systematically building a Petri net model of a manufacturing system. As invariants play a central role in studying the qualitative properties of Petri nets we give a procedure for computing the invariants of the nets. In Section 4, we apply these procedures to some illustrative examples. In Section 5, we discuss a higher level Petri net, called the coloured Petri net, and apply it to the modeling and analysis of an FMS. Coloured Petri nets provide a concise representation of large Petri nets.

2.0 FUNDAMENTAL CONCEPTS AND NOTATIONS OF PETRI NETS

Petri nets were originally developed by Carl Adam Petri (1962) to model information flow in systems. Since then a large body of knowledge has been discovered on both the nature and the application of Petri nets. In this section we give the basic definitions and concepts of Petri nets and explain them by means of examples. We also give some properties of Petri nets that are relevant to manufacturing systems.

2.1 Basic Definitions

The static aspects of a Petri net are defined as follows.

Definition: A marked Petri net is a 5-tuple (P, T, I, O, M_o), where

(i) $P = (p_1, p_2, \ldots, p_n)$ is a finite set of places;

(ii) $T = (t_1, t_2, \ldots, t_m)$ is a finite set of transitions;

(iii) $P \cup T \neq \phi$ and $P \cap T = \phi$;

(iv) I and O are, respectively, the input function and the output function that assign a nonnegative integer to each pair $(p,t) \in P \times T$;

(v) M_o is an initial marking that associates a nonnegative integer to each place p_i.

A Petri net can be represented by a bipartite directed graph with two kinds of nodes, places, and transitions. Places are represented by circles and transitions are represented by bars. A directed arc is drawn from a place p_i to a transition t_j if $I(p_i,t_j) \neq 0$. The value $I(p_i,t_j)$ gives the weight of the arc from p_i to t_j. The directed arc (p_i,t_j) is labelled by $I(p_i,t_j)$ if $I(p_i,t_j) > 1$. The label 1 is omitted. In a similar way, the output function is represented by a directed arc from the transitions to places. The marking

M_o is represented by drawing $M_o(p)$ dots in each place p. The presence of a certain number of dots, called tokens, in a place may be interpreted as the number of data items or conditions associated with the place. The initial marking M_o may be considered as the initial state of the system.

In the above definition it is assumed that the capacity of all places is infinite. If the capacity K(p) is finite, then $M_o(p) \leq K(p)$ in item (v) of the Definition.

Example 1. Let us consider the case of a transfer line consisting of two machines M_1 and M_2 and a buffer B of finite capacity K(B) shown in Figure 1(a). The flow control mechanism is such that a new job enters the system as soon as M_1 is free. Figure 1(b) depicts the Petri net model of the system.

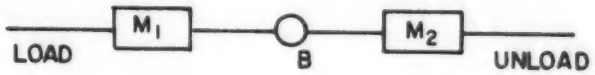
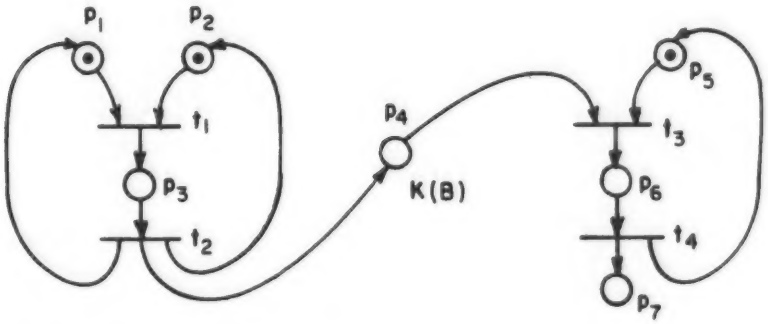


Figure 1(a). A transfer line with two machines and one buffer.



p_1 = job waiting for processing
p_2 = M_1 ready to process
p_3 = job being processed on M_1
t_1 = process begins on M_1
t_2 = process ends on M_1

p_4 = buffer B with capacity K(B)
p_5 = M_2 ready to process
p_6 = job being processed on M_2
p_7 = finished product
t_3 = process begins on M_2
t_4 = process ends on M_2

Figure 1(b). A Petri net model of the transfer line.

For this example, it should be noted that for the transition, say t_1, $I(p_1,t_1) = I(p_2,t_1) = 1$ and $O(p_3,t_1) = 1$, and the rest of the values are all zero.
The net has seven places and four transitions. The places represent various conditions in the system, whereas transitions represent the start or finish of operation. The initial marking M_o may be represented as a column vector $M_o = [1\ 1\ 0\ 0\ 1\ 0\ 0]^T$ where T denotes the transpose of a matrix and the ith entry in M_o is the number of tokens in place p_i.

To study the dynamic aspects of a Petri net we need to know the rules that govern the change in the marking as the transition occurs. Such rules are called transition rules. For $K(p) = \infty$, the rules are as follows

Definition: Let $G = (P, T, I, O, M_o)$ be a Petri net.

(i) A transition $t \in T$ is said to be enabled or firable at M_o if

$$M_o(p) \geq I(p,t), \text{ for all } p \in P \tag{1}$$

(ii) A transition $t \in T$ can fire when it becomes enabled.

(iii) When an enabled transition t_j fires, a new marking M is reached according to the equation

$$M(p) = M_o(p) + O(p,t_j) - I(p,t_j), \text{ for all } p \in P \tag{2}$$

The marking M is said to be immediately reachable from M_o.

Equation (2) can be written in the vector-matrix form:

$$M = M_o + CE_j \tag{3}$$

where C is an $(n \times m)$ incidence matrix such that

$$c_{ij} = O(p_i,t_j) - I(p_i,t_j)$$

and the elementary transition E_j is an m-vector whose entries are zero except the jth entry which is equal to one. The element c_{ij} represents the token change in place p_i when transition t_j fires.

A marking M' is said to be reachable from a marking M if there exists a sequence of firing that transforms M to M'. The set of all reachable markings from M is denoted by R(M).

For the system shown in Figure 1(b), the only transition that is enabled at M_o is t_1. The firing of t_1 gives the marking $M = [0\ 0\ 1\ 0\ 1\ 0\ 0]^T$. At this marking t_2 is enabled, the firing of which enables t_1 and t_3 and so on.

The transition rule given in the above definition applies to the Petri net with infinite capacity. For the Petri net with finite capacity, there is an additional condition for a transition to be enabled so that the number of tokens in each of the output places will not exceed its capacity after firing. The modified rule can be expressed by the equation

$$I(p,t) \leq M_o(p) \leq K(p) - O(p,t), \text{ for all } p \in P.$$

2.2 Concurrency and Conflict

In this section we shall discuss two fundamental concepts of the Petri net theory: concurrency and conflict. Consider the Petri net shown in Figure 2. It consists of six places and five transitions. For the marking shown in the figure transitions t_1 and t_3 are enabled and can both fire. Two such transitions are said to be concurrent. Transitions t_1 and t_3 are causally

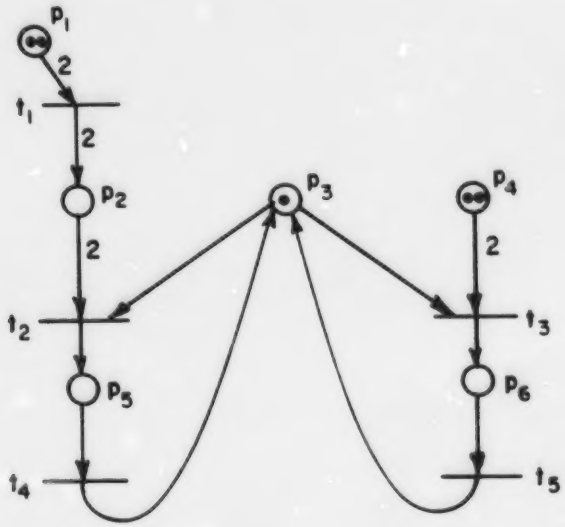


Figure 2. A Petri net depicting concurrency.

independent and one transition can fire before, after, or in parallel with the other. Formally, this concept can be defined in the following way.

Definition: Two transitions t_1 and t_2 are concurrently enabled at a marking M if for all $p \in P$

$$I(p,t_1) + I(p,t_2) \leq M(p) \leq K(p) - O(p,t_1) - O(p,t_2) \qquad (4)$$

It should be noted that when $K(p) = \infty$, the right side inequality in Eq. (4) is redundant.

Suppose the transition t_1 is fired. The firing of t_1 removes two tokens from p_1 and deposits two tokens in p_2. At this new marking the transitions t_2 and t_3 are enabled. However, they both cannot fire because the firing of one disables the other. The transitions t_2 and t_3 are said to be in conflict. Since either t_2 or t_3 can fire we have to make a choice as to which one should be fired first. This signifies the nondeterminism inherent in Petri nets. The conflict (choice) is formally defined as the following.

Definition: Two transitions t_1 and t_2 are in conflict at a marking M if both are enabled at M, but not concurrently enabled.

Consider again the marked Petri net shown in Figure 2 and let the marking $M = [0\ 2\ 0\ 0\ 0\ 1]^T$ be reached through the firing of transitions t_1 and t_3. There are two sequential ways of arriving at this marking. The first sequence is the one in which t_3 was fired first and then t_1, while the second sequence is where t_1 was fired first and then t_3 was fired by resolving the conflict between transitions t_2 and t_3, in favour of t_3. There is no way to tell as to which sequence did really occur. Such a situation is called confusion. This phenomenon is quite common and arises whenever concurrency and conflict 'overlap'.

2.3 State Equation

Let M_{k-1} denote the state of the Petri net at the kth firing and let v_k be an $(m \times 1)$ column vector of ones and zeroes. The jth entry of vector v_k is one if and only if the transition t_j is to be fired at the kth firing. Then the state M_k resulting from the kth firing is given by

$$M_k = M_{k-1} + Cv_k, \quad k = 1,2, \ldots \tag{5}$$

This equation follows easily from Eq. (3) and the definition of the incidence matrix C. The vector v_k is called the firing or control vector. By applying Eq. (5) to $k = 1,2,\ldots q$ and combining the resulting equations we obtain

$$M_q = M_o + C \sum_{k=1}^{q} v_k$$

or simply

$$M = M_o + CF \tag{6}$$

where $M = M_q$ and $F = \sum_{k=1}^{q} v_k$ is an m-vector of nonnegative integers called the firing count vector (Murata 1984). The jth entry of F denotes the number of times the transition t_j must fire to reach the marking M. Since $M_k \geq 0$, it follows from Eq. (5) that the sequence of firing v_k must satisfy

$$M_{k-1} + Cv_k \geq 0 \text{ for each } k \tag{7}$$

Given the sequence $(v_1, v_2 \ldots v_q)$ of firing vectors we can determine the reachable marking M from Eq. (6). In other words, any reachable marking satisfies Eq. (6). However, the converse is not true; that is, an F satisfying Eq. (6) for a given M does not imply that M is reachable from M_o. A Petri net showing this is given in Figure 3.

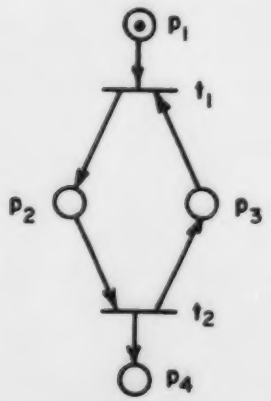


Figure 3. A Petri net showing that the existence of F does not imply reachability.

For the Petri net shown in Figure 3 we have

$$M_o = [1\ 0\ 0\ 0]^T,\ C^T = \begin{bmatrix} -1 & 1 & -1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

Let $M = [0\ 0\ 0\ 1]^T$. For this value of M, it is easy to check that $F = [1\ 1]^T$ is a solution of Eq. (6). However, M is not a reachable marking from M_o since none of the transitions are enabled at M_o. It should be noted that $F = [1\ 1]^T$ is the unique solution of Eq. (6).

The firing count vector F can be found for a reachable marking M by solving Eq. (6). Let C^* denote the pseudo inverse of C. Then F is given by:

$$F = C^*\Delta M + (I_m - C^*C)z \qquad (8)$$

when $\Delta M = M - M_o$, I_m is the unit $(m \times m)$ matrix and z is an arbitrary m-vector. When the matrix C is of rank m, the pseudo inverse $C^* = (C^TC)^{-1}C^T$. Substituting this value of C^* into Eq. (8) we obtain

$$F = (C^TC)^{-1}\ C^T\Delta M$$

For the Petri net shown in Figure 3, the rank of the matrix C is 2 and hence

$$C^* = (C^TC)^{-1}\ C^T$$

$$= \frac{1}{5}\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -1 & 1 & -1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} -3 & 1 & -1 & 2 \\ -2 & -1 & 1 & 3 \end{bmatrix}$$

It should be mentioned that the knowledge of F does not provide the sequence of control vectors v_k, $k = 1,2,\ldots$, satisfying Eq. (7), but merely gives the number of times each transition must be fired to reach the marking M from the initial marking M_o.

2.4 Some Properties of Petri Nets

In this section we shall study some properties of Petri nets that have relevance for studying the dynamic behavior of FMS.

Definition: Safeness and boundedness. A place p_i of a Petri net is said to be bounded if there exists a positive number k such that

$$M(p_i) \leq k, \text{ for all } M \in R(M_o)$$

If $k = 1$, we say that the place is safe. The Petri net is said to be bounded (safe) if each place of the net is bounded (safe).

Definition: Deadlock free. A Petri net is said to be deadlock free if, for all $M \in R(M_o)$, at least one transition is enabled.

Definition: Properness. A Petri net is said to be proper or reversible if, for all $M \in R(M_o)$, $M_o \in R(M)$.

The above mentioned properties have simple and useful interpretations for an FMS. In the modeling of an FMS by Petri nets, the operations and resources will be represented by places in the Petri net. Safeness of an operation place means that there is no request made for the execution of an operation if it is already in progress. If the resource place is safe or bounded, it implies that the resources are limited. For example, if buffer is a resource place, then the buffer size is finite and can be obtained by finding the bound representing the buffer place. Deadlocks occur if there are insufficient resources for the processing of the operations. Knowing that deadlock exists, corrective measures may be taken to rectify it.

Properness implies that the system will eventually reinitialize itself from any current state. If an FMS is not proper, then a manual intervention is necessary to bring the system to its initial state.

Figures 4(a) - 4(d) depict the various properties discussed above.

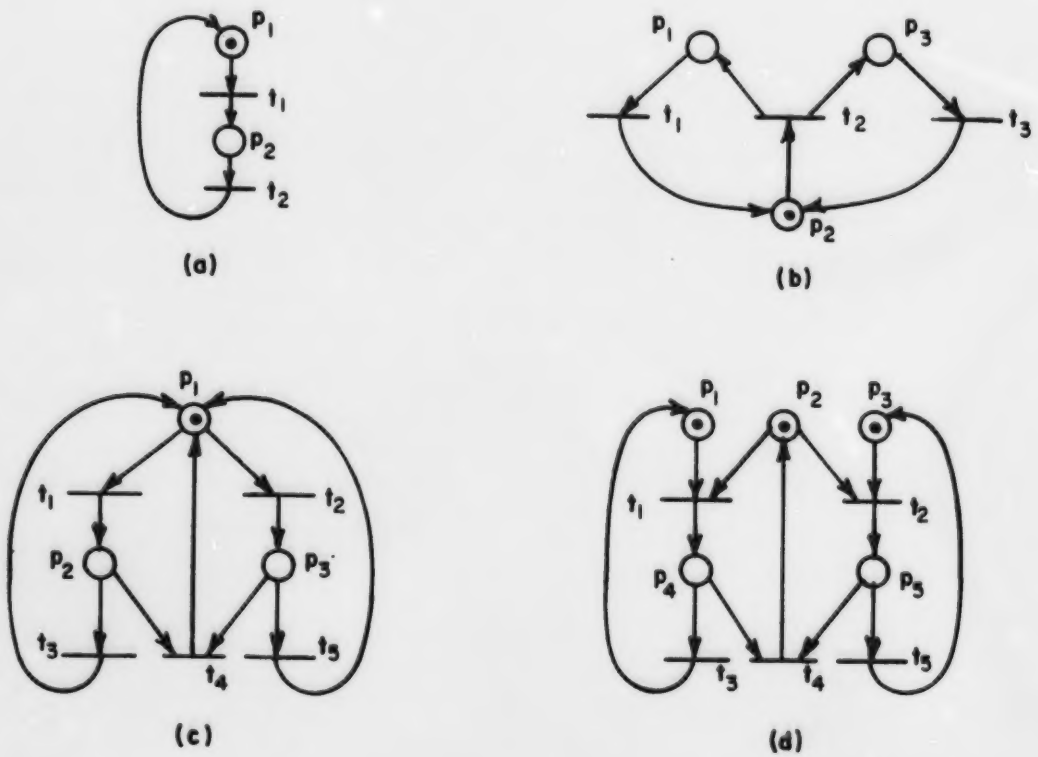


Figure 4. (a) A Petri net that is safe, deadlock free, and proper.
(b) A Petri net that is not bounded, deadlock free, and not proper.
(c) A Petri net that is safe, deadlock free, and proper; transition t_4 can never be fired.
(d) A Petri net that is safe, deadlocked, and not proper.

Another property of net that is useful is called liveness.

Definition: Liveness. A transition t_j of a Petri net is said to be live if, for all $M \in R(M_o)$, there exists a sequence of transition firings such that t_j is enabled. A Petri net is said to be live if all its transitions are live.

It should be noted that a live net is deadlock free. The Petri net shown in Figure 4(c) is deadlock free, but not live. Transition t_4 is dead; that is, it can never be fired.

2.5 Petri Net Invariants

Many dynamical properties of a Petri net can be analyzed by using p-invariants (place invariants) and t-invariants (transition invariants). They are obtained as solutions of linear system of equations.

In the sequel we shall consider only those nets that are pure or self-loop free. A net is said to be pure if there exists no pair (p_i,t_j) such that $I(p_i,t_j) \neq 0$ and $O(p_i,t_j) \neq 0$. This is not a restriction, since any net that is not pure can be transformed into an equivalent pure net by introducing pairs of dummy places and transitions.

A column vector x indexed by the set of places will be called a p-vector and a column vector y indexed by the set of transitions will be called a t-vector. The marking M is therefore a p-vector and the firing vector v_k and the firing count vector F are t-vectors. A p-vector x is said to be p-invariant if

$$x^T C = 0 \tag{9}$$

Similarly, a t-vector y is said to be t-invariant if

$$Cy = 0 \tag{10}$$

The invariants x and y, according to the above definitions, could have real numbers as components. However, we shall be mainly concerned with invariants whose entries are integers. The subset of places (transitions) corresponding to nonzero entries of x(y) is called the support and is denoted by $\|x\|(\|y\|)$. A support is called minimal if no proper subset of the support is another support. A vector x(y) having minimal support is called a proper p-invariant (t-invariant).

Let M be a marking reachable from the initial marking M_o. Then M is given by

$$M = M_o + CF$$

Premultiplying the above equation by the transpose of the p-invariant x, we have

$$x^T M = x^T M_o + x^T CF = x^T M_o \tag{11}$$

which implies that the weighted sum of tokens is a constant, the weights being given by the components of the p-invariant x. Note that, if x(p) = 1 for all places, the weighted sum is the total number of tokens in the net.

Equation (11) gives a necessary condition for a marking M reachable from the initial marking. This means that if there exists a p-invariant x such that

$$x^T M \neq x^T M_o \tag{12}$$

then M is not reachable from M_o.

For the Petri net shown in Figure 3, it can be seen that $x = [1\ 2\ 1\ 1]^T$ is a p-invariant. With $M = [0\ 0\ 0\ 1]^T$, $M_o = [1\ 0\ 0\ 0]^T$ we have $x^T M = 1 = x^T M_o$. Hence Eq. (11) is satisfied, however M is not reachable from M_o. The support of the p-invariant x is $\|x\| = (p_1, p_2, p_3, p_4)$. This is not the minimal support since (p_1, p_2, p_4) and (p_2, p_3), respectively, are supports of the p-invariants $x' = [1\ 1\ 0\ 1]^T$ and $x'' = [0\ 1\ 1\ 0]^T$. This net has two linearly independent p-invariants x' and x".

Let r denote the number of linearly independent p-invariants and let B_p denote the $(n \times r)$ matrix formed by these invariants. Then, for $M \in R(M_o)$, we have

$$B_p^T M = B_p^T M_o$$

or

$$B_p^T \Delta M = 0 \tag{13}$$

where $\Delta M = M - M_o$. This follows from the fact that each column of B_p satisfies Eq. (11). Similarly, if

$$B_p^T \Delta M \neq 0 \tag{14}$$

then M is not reachable from M_o. Equation (14) can be used for fault detection.

For the Petri net shown in Figure 3, B_p^T is given by

$$B_p^T = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

The boundedness of a Petri net can be established by the result stated below.

Result 1. A Petri net is bounded if there exists a p-invariant x for which all entries are strictly positive.

We now turn our attention to the t-invariants. Let M be reachable from the marking M_o. Then $M = M_o + CF$. If F is a t-invariant we have $M = M_o$. This implies that a t-invariant gives the number of times the transitions must be fired in order that a particular marking may be reproducible. The following result gives a necessary condition for properness.

Result 2. A Petri net is not proper if there exists no t-invariant y such that $y \geq 0$ and $y \neq 0$.

The net in Figure 3 is not proper since $y = 0$ is its only t-invariant.

Example 2. Consider the Petri net shown in Figure 4(d). For this net the incidence matrix C is given by

$$C = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & -1 \end{bmatrix}$$

Solving Eqs. (9) and (10) it is easy to see that $x = [x_1\ 0\ -x_1\ x_1\ -x_1]^T$ is a p-variant and $y = [y_1\ -y_1\ y_1\ 0\ -y_1]^T$ is a t-invariant. The scalars x_1 and y_1 can assume any real values. However, as mentioned before we restrict them to integer values only. The matrix C has rank 4, which implies that there is only one linearly independent solution to each of Eq. (9) and (10). Since there is no t-invariant y satisfying $y \geq 0$ and $y \neq 0$ it follows from Result 2 that the net is not proper. From Eq. (11) it follows that

$$M(p_1) - M(p_3) + M(p_4) - M(p_5) = M_o(p_1) - M_o(p_3) + M_o(p_4) - M_o(p_5) = 0 \quad (15)$$

From the t-invariant, for $y_1 > 0$, we conclude that transitions t_1 and t_3 may be fireable. For the given marking M_o, indeed such is the case. Firing transitions t_1 and t_3 we have $M(p_1) = 1$, $M(p_2) = 0$ and $M(p_4) = 0$. Using this partial knowledge of the reachable marking and Eq. (15) we get

$$M(p_3) + M(p_5) = 1 \quad (16)$$

Since $M(p_i) \geq 0$, it follows that either $M(p_3) = 0$ and $M(p_5) = 1$ or $M(p_3) = 1$ and $M(p_5) = 0$. In the latter case, it is easy to check that no transition is enabled. Also the former case reduces to the latter case after firing transition t_5. From this we conclude that there is a marking $M = [1\ 0\ 1\ 0\ 0]^T$ reachable from M_o such that no transition is enabled. Hence the system is deadlocked.

Example 3. Consider the Petri net of Figure 4(c). The incidence matrix C of this net is given by

$$C = \begin{bmatrix} -1 & -1 & 1 & 1 & 1 \\ 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & -1 \end{bmatrix}$$

This matraix has rank 3. hence, $x = 0$ is the only p-invariant and there are $(5 - 3) = 2$ linearly independent t-invariants. Solving Eq. (10) we see that the two t-invariants are given by $y' = [1\ 0\ 1\ 0\ 0]^T$ and $y'' = [0\ 1\ 0\ 0\ 1]^T$. Since $x = 0$ is the only p-invariant no information about the system behavior can be derived from it. However, with the information that transition t_4 is dead, we can remove the fourth column from C and obtain the reduced incidence matrix $\hat{C}$ as

$$\hat{C} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Solving Eq. (9) with $\hat{C}$ as the incidence matrix, we find that $x = (x_1, x_1, x_1)^T$ is a p-invariant. From Eq. (11) we thus obtain

$$M(p_1) + M(p_2) + M(p_3) = 1 \qquad (17)$$

Since $M(p_i) \geq 0$, Eq. (17) shows that for all marking reachable from M_o, one of the places p_i has a token and the others have none. Hence, for all $M \in R(M_o)$ at least one transition t_i is enabled and the system is deadlock free.

The Petri net of Figure 4(c) has three distinct markings given by $M_o = [1\ 0\ 0]^T$, $M_1 = [0\ 1\ 0]^T$ and $M_2 = [0\ 0\ 1]^T$.

3.0 MODELING AND ANALYSIS OF FMS

In this section we discuss the procedures for modeling and analysis of flexible manufacturing systems using Petri nets.

3.1 Modeling of FMS

Places, transitions and tokens defining the Petri nets are mathamatical objects. To model a particular class of systems these objects must be interpreted in terms of the entities of the system being modeled. For an FMS, the basic elements are the operations and the resources required to perform these operations. By an operation we mean a single stage of the process which is performed independently from the other parts of the system. Examples of operations are the loading and unloading of parts on machines, processing of parts on machines, etc. Each operation requires a set of resources such as raw materials, tools, robots, machines, etc. A sequence of operations is generally required to process each part type and each operation may be performed on more than one machine.

To model an FMS by Petri net, we identify the places with resources or operations. A token present in the resource place indicates that the resource is available. Similarly, a token present in the operation place indicates that the operation is in progress. The transitions are identified with the start or end of operations. The Petri net model is synthesized in the following manner.

(i) Represent each operation with the set of resources required by the Petri net. Generally, there will be more than one Petri net model for each operation, since alternative processing is allowed.

(ii) Obtain the Petri net model for each operation by combining the models in step (i) along common places.

(iii) Obtain the Petri net model for each part type as the union of Petri net models, corresponding to the operations belonging to the specific part type.

(iv) Finally, construct the Petri net model of an FMS as the union of Petri net models of all part types.

The union of two nets is defined in an obvious way. Let $G_1 = (P_1, T_1, I_1, O_1)$ and $G_2 = (P_2, T_2, I_2, O_2)$ be two Petri nets such that $T_1 \cap T_2 = \phi$. The union of

G_1 and G_2 is denoted by $G = (P,T,I,O)$ where $P = P_1 \cup P_2$, $T = T_1 \cup T_2$, $I = I_1 \cup I_2$ and $O = O_1 \cup O_2$. Figure 1(b) is the union of two subnets, one net representing the processing on machine M_1 and the other processing on machine M_2. The buffer place p_4 is common to both the subnets.

3.2 Analysis of FMS

The p- and t-invariants have played significant roles in analyzing the Petri nets. For large systems, the determination of solutions to the linear homogeneous systems of Eqs. (9) and (10) is not easy. However, if the Petri net model is constructed according to the procedure outlined in the previous section, there is a systematic method for constructing the p-invariants of the net from the p-invariants of its subnets. Let x' and x'' be the p-invariants of the subnets G_1 and G_2, such that $x'(p) = x''(p)$ for all $p \in P_1 \cap P_2$. Further, let $\bar{x}''$ denote the vector obtained from x'' by deleting the elements common to x' and x''. Then the following result holds (Narahari et al. 1985a).

Theorem. Let G_1 and G_2 be two pure Petri nets such that their union G is also pure. Also, let $T_1 \cap T_2 = \phi$. Then

$$x = \begin{bmatrix} x' \\ \bar{x}'' \end{bmatrix}$$

is a p-invariant of G if and only only if x' and x'' are respectively the p-invariants of G_1 and G_2.

Example 4. Consider the Petri net shown in Figure 1(b). The net G_1 consists of four places $(p_1,\ldots,p_4)$ and two transitions (t_1, t_2). By solving Eq. (9) we find that $x' = [x_1\ x_2\ (x_1+ x_2)\ 0]^T$ is a p-invariant. Similarly, for the second subnet G_2 consisting of four places $(p_4,\ldots,p_7)$ and two transitions $t_3, t_4)$, $x'' = [x_4\ x_5\ (x_4+ x_5)\ x_4]^T$is its p-invariant. Since $p_4$is the only common place $x'(p_4) = x''(p_4)$ yields $x_4 = 0$. By applying the above theorem, the p-invariant of G is given by

$$x = [x_1\ x_2\ (x_1+ x_2)\ 0\ x_5\ x_5\ 0]^T$$

As there are three independent variables x_1, x_2 and x_5 in the above p-invariant, there exist three linear independent p-invariants of the Petri net of Figure 1(b). These are given by

$$x_a = [1\ 0\ 1\ 0\ 0\ 0\ 0]^T,\ (x_1 = 1,\ x_2 = x_5 = 0)$$

$$x_b = [0\ 1\ 1\ 0\ 0\ 0\ 0]^T,\ (x_2 = 1,\ x_1 = x_5 = 0)$$

$$x_c = [0\ 0\ 0\ 0\ 1\ 1\ 0]^T,\ (x_5 = 1,\ x_1 = x_2 = 0)$$

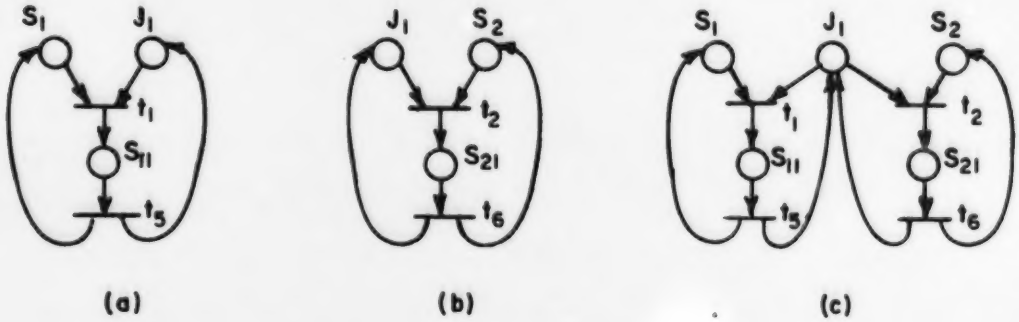
4.0 ILLUSTRATIVE EXAMPLES

In this section we discuss a few examples to illustrate the use of p-invariants for studying the qualitative properties of manufacturing systems.

4.1 A Manufacturing System With Two Machines and Two Part Types

As our first example we consider the case of only two work stations S_1 and S_2 and two part types J_1 and J_2. Each part type goes through a single stage of processing which can be performed on either S_1 or S_2. It is assumed that a part is made available to the system as soon as a work station is available. Figures 5(a) - 5(c) show the Petri net model of the processing of J_1 on S_1, S_2, and either S_1 or S_2, respectively. Figure 5(d) depicts the Petri net model of the whole system. The interpretation of the various places and transitions is as follows (i,j = 1,2).

S_i = work station S_i
J_j = part type j
S_{ij} = processing of part type j on work station S_i
t_1 - t_4 = start of processing
t_5 - t_8 = end of processing



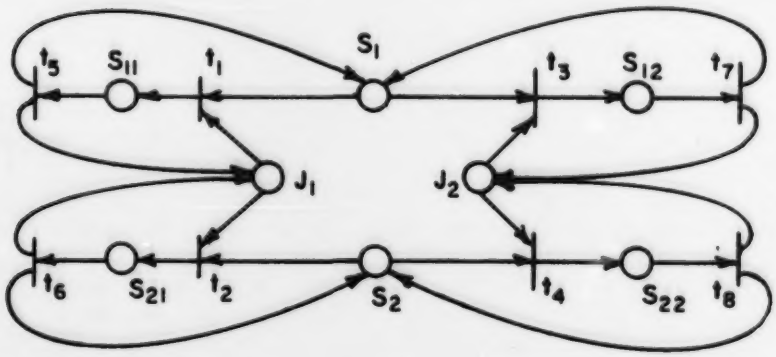


Figure 5. (a) Petri net model of processing J_1 on S_1.
(b) Petri net model of processing J_1 on S_2.
(c) Petri net model of processing J_1 on either S_1 or S_2.
(d) Petri net model of the whole system.

The p-invariants of the nets shown in Figure 5(a) and Figure 5(b), corresponding to the places (S_1,J_1,S_{11}) and (S_2,J_1,S_{21}), are respectively given by $[x_1\ x_3\ (x_1 + x_3)]^T$ and $[x_2\ x_4\ (x_2 + x_4)]^T$. Since J_1 is common to both the nets, by setting $x_4 = x_3$ and applying the theorem we obtain $[x_1\ x_2\ x_3\ (x_1 + x_3)\ (x_2 + x_3)]^T$ as the p-invariants for the net of Figure 5(c) corresponding to the places $(S_1,S_2,J_1,S_{11},S_{22})$. Similarly, the p-invariants of Figure 5(d) can be found and are given by

$$x = [x_1\ x_2\ x_3\ x_4\ (x_1+ x_3)\ (x_2+ x_3)\ (x_1+ x_4)\ (x_2+ x_4)]^T \qquad (18)$$

where the place ordering is taken as $(S_1,S_2,J_1,J_2,S_{11},S_{21},S_{12},S_{22})$.

Let the initial marking M_o be given by $M_o(S_1) = M_o(S_2) = M_o(J_1) = M_o(J_2) = 1$ and $M_o(S_{ij}) = 0, i,j = 1,2$. Substituting these values and Eq. (18) into Eq. (11) we obtain the following four equations.

$$M(S_1) + M(S_{11}) + M(S_{12}) = 1 \qquad (19)$$

$$M(S_2) + M(S_{21}) + M(S_{22}) = 1 \qquad (20)$$

$$M(J_1) + M(S_{11}) + M(S_{21}) = 1 \qquad (21)$$

$$M(J_2) + M(S_{12}) + M(S_{22}) = 1 \qquad (22)$$

Equation (19) says that amongst the places S_1, S_{11}, and S_{12} only one of them has a token count of one. This implies that either S_1 is ready to process or is processing J_1 or J_2, but not both. Similar conclusions can be drawn from the other equations. As the places can have at most one token, the net is safe.

Using Eqs. (19)-(22) we can show that the system is deadlock free. Assume that the system is deadlocked, that is, there is a reachable marking where no transition is enabled. Since transitions t_5, t_6, t_7 and t_8 are not enabled we have $M(S_{ij}) = 0$, $i,j = 1,2$. Substituting these values into Eqs. (19)-(22) we get $M(S_1) = M(S_2) = M(J_1) = M(J_2) = 1$. Hence, transitions t_1, t_2, t_3, and t_4 are enabled. This contradicts our assumption that no transition is enabled and hence the system is deadlock free.

The incidence matrix C is (8 × 8) and is given by

$$C = \begin{array}{c} \\ S_1 \\ S_2 \\ J_1 \\ J_2 \\ S_{11} \\ S_{21} \\ S_{12} \\ S_{22} \end{array} \begin{array}{c} \begin{array}{cccc|cccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 \end{array} \\ \left[\begin{array}{cccc|cccc} -1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 & 1 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 \\ \hline 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{array}\right] \end{array} \qquad (23)$$

4.2 A Simple Example of a Manufacturing System With a Deadlock

Consider a set of two machines S_1 and S_2 processing two part types J_1 and J_2. The processing sequence for part type J_1 is $S_1 \rightarrow S_2$ and that of J_2 is $S_2 \rightarrow S_1$. We assume that the machine set-up times and transportation times are negligible and that there is no buffer storage available. We further assume that there is one pallet for each part type and that a part is let into the system whenever the pallet of its type is available. Figures 6(a) and 6(b) represent, respectively, the Petri net models of the processing of parts of type J_1 and J_2. The union of the graphs shown in Figures 6(a) and 6(b) then gives the Petri net of the whole system. The interpretation of the places and transitions is as follows (i,j,k = 1,2).

S_i = machine S_i

P_j = pallet of type j

S_{ikj} = work station S_i processing the kth operation on part type J_j

W_{ji} = part type J_j waiting for work station S_i after the first operation

$t_1 - t_4$ = start of processing

$t_5 - t_8$ = end of processing

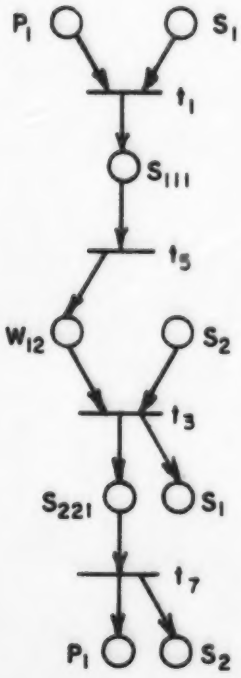


Figure 6(a). Petri net model of the processing of part type J_1.

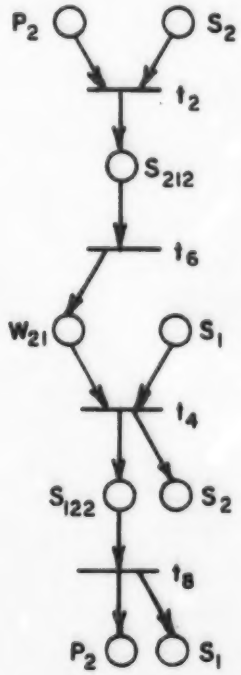


Figure 6(b). Petri net model of the processing of part type J_2.

The p-invariants for the Petri nets of Figures 6(a) and 6(b) can be easily calculated and are shown in Table 1 along with p-invariants of the whole

net. The p-invariants for the system are obtained from the two subsystems by writing $x_2' = x_1$ and $x_1' = x_2$.

S_1	S_2	P_1	S_{111}	W_{12}	S_{221}
x_1	x_2	x_3	x_1+x_3	x_1+x_3	x_2+x_3

(a)

S_2	S_1	P_2	S_{212}	W_{21}	S_{122}
x_1'	x_2'	x_4	$x_1'+x_4$	$x_1'+x_4$	$x_2'+x_4$

(b)

S_1	S_2	P_1	P_2	S_{111}	S_{212}	W_{12}	W_{21}	S_{221}	S_{122}
x_1	x_2	x_3	x_4	x_1+x_3	x_2+x_4	x_1+x_3	x_2+x_4	x_2+x_3	x_1+x_4

(c)

Table 1. (a) Place invariants of Figure 6(a).
(b) Place invariants of Figure 6(b).
(c) Place invariants of the whole system.

Let the initial marking be given by $M_o(S_1) = M_o(S_2) = M_o(P_1) = M_o(P_2) = 1$ and $M_o(S_{111}) = M_o(S_{212}) = M_o(W_{12}) = M_o(W_{21}) = M_o(S_{221}) = M_o(S_{122}) = 0$. Then for any marking M reachable from M_o we obtain the following four equations from the p-invariants of the system.

$$M(S_1) + M(S_{111}) + M(W_{12}) + M(S_{122}) = 1 \quad (24)$$

$$M(S_2) + M(S_{212}) + M(W_{21}) + M(S_{221}) = 1 \quad (25)$$

$$M(P_1) + M(S_{111}) + M(W_{12}) + M(S_{221}) = 1 \quad (26)$$

$$M(P_2) + M(S_{212}) + M(W_{21}) + M(S_{122}) = 1 \quad (27)$$

By firing the transitions t_1, t_2, t_5 and t_6 we arrive at the marking where $M(W_{12}) = M(W_{21}) = 1$. From Eqs. (24)-(27) or otherwise we find that the rest of the places have no tokens. This implies that at this marking no transition is enabled, and hence we have a deadlock. This deadlock can be avoided if a buffer to store the waiting parts is provided. This can be easily seen by constructing the Petri net model for this system and its p-invariants. We mention that in this model S_1 and S_2, respectively, belong to the output places of transitions t_5 and t_6 rather than t_3 and t_4.

It should be noted that since the number of tokens in each place is 0 or 1, the net is safe and bounded. However, it is not proper because it is deadlocked.

4.3 A Manufacturing System With Three Machines and Three Part Types

As our last example we consider the case of three machines and three part types. The parts routing is fixed and is shown in Table 2.

Part \ Oper	1	2	3
J_1	M_2	M_3	
J_2	M_1	M_2	M_3
J_3	M_1	M_2	

Table 2. Operation sequence for the parts.

We assume, as before, that the set-up times and transportation times are negligible and that there is only one pallet for each part type. We further assume that a part is let into the system whenever there is a pallet of its type available. Figures 7(a) to 7(c) show the Petri net models of part types J_1, J_2 and J_3, respectively. The union of these graphs then gives the Petri net of the whole system. The interpretation of the various places and transitions is similar to that of the previous example. The place $W_{jk}(j,k = 1,2,3)$, for example, means that the part type j is waiting for the kth operation.

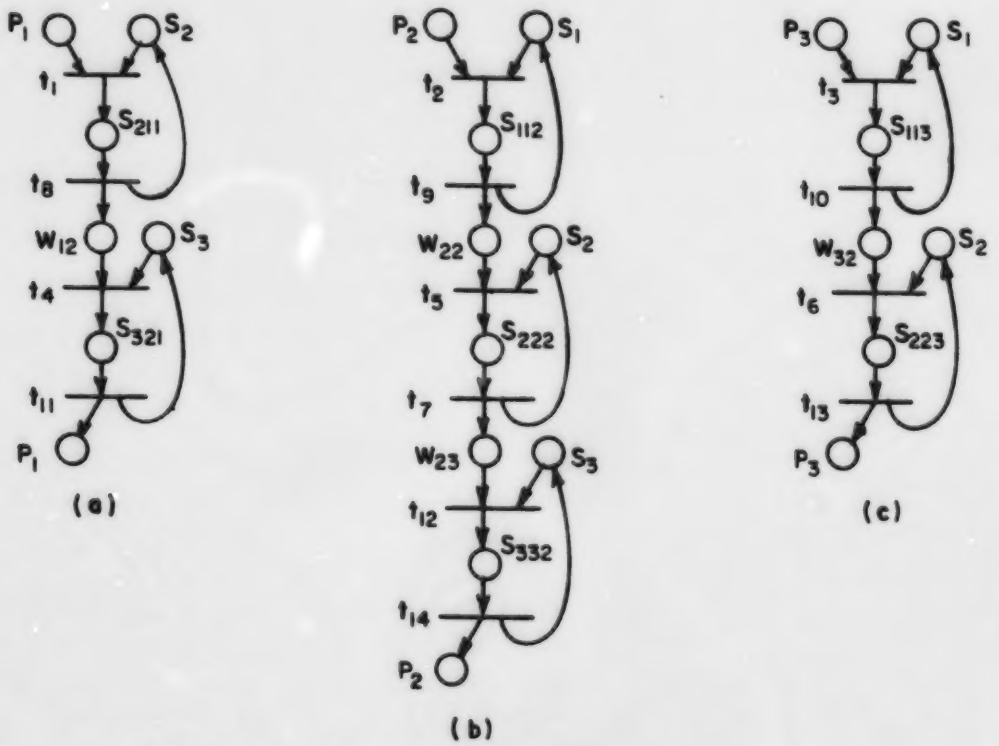


Figure 7. (a) Petri net model of part type J_1.
(b) Petri net model of part type J_2.
(c) Petri net model of part type J_3.

The p-invariants of the Petri net of the whole system can be found using the general procedure. As there are six independent places, three corresponding to the machines and three corresponding to the part types, the number of linearly independent p-invariants is six. Table 3 depicts the p-invariants of the complete system.

Place	S_1	S_2	S_3	P_1	P_2	P_3	S_{211}	S_{112}	S_{113}
Component	x_1	x_2	x_3	x_4	x_5	x_6	$x_2+ x_4$	$x_1+ x_5$	$x_1+ x_6$
Place	W_{12}	W_{22}	W_{32}	S_{321}	S_{222}	S_{223}	W_{23}	S_{332}	
Component	x_4	x_5	x_6	$x_3+ x_4$	$x_2+ x_5$	$x_2+ x_6$	x_5	$x_3+ x_5$	

Table 3. The p-invariants of FMS with three machines and two part types.

The p-invariants in this example and the previous ones can be easily determined by inspection, keeping in mind the interpretation of the various places. Consider, for example, the place S_{211}. This place signifies the fact that the machine S_2 is performing the first operation on part type J_1. As this place is holding the machine S_2 and the part type J_1, the element in the p-invariant must reveal this fact. Thus, if the element in the p-invariant corresponding to S_2 and J_1 is x_2 and x_4 respectively, the element corresponding to S_{211} must be $x_2 + x_4$. Other elements shown in Table 3 can be similarly checked.

Let us assume that initially all machines and all pallets are available. Using the p-invariants we have the following six equations.

$$M(S_1) + M(S_{112}) + M(S_{113}) = 1 \tag{28}$$

$$M(S_2) + M(S_{211}) + M(S_{222}) + M(S_{223}) = 1 \tag{29}$$

$$M(S_3) + M(S_{321}) + M(S_{332}) = 1 \tag{30}$$

$$M(P_1) + M(S_{211}) + M(W_{12}) + M(S_{321}) = 1 \tag{31}$$

$$M(P_2) + M(S_{112}) + M(W_{22}) + M(S_{222}) + M(W_{23}) + M(S_{332}) = 1 \tag{32}$$

$$M(P_3) + M(S_{113}) + M(W_{32}) + M(S_{223}) = 1 \tag{33}$$

From Eqs. (28) to (33) it can be easily seen that the Petri net of the whole system is safe, bounded, and deadlock free.

5.0 COLOURED PETRI NETS

For small problems, as is evidenced by the examples of the previous section, the description of an FMS by Petri net is undoubtedly simple and easily manageable. However, for larger problems the Petri net description becomes unwieldy. To alleviate this difficulty, a higher level net called coloured Petri net can be used. In this net a colour is attached to each token indicating its identity. Moreover each place and each transition has a set of colours attached to them.

Before defining the coloured Petri net formally, we shall use the problem of Section 4.1 as a vehicle for explaining the concepts of coloured Petri nets. For this problem, we have two part types J_1, J_2 and two machines S_1 and S_2. A token in place $J_1(J_2)$ means that a part of type $J_1(J_2)$ is available for processing. This suggests that we could represent the two places by a single place J containing tokens of the two types J_1 and J_2. This distinction can be made by saying that the place J has the two distinct colours J_1 and J_2. In the same way, the places S_1 and S_2 (representing the machines) and S_{11}, S_{21}, S_{12} and S_{22} (representing the processing) can be combined, respectively, into places S and P. The place S has two colours and the place P has four colours. The transitions t_1 to t_4 and t_5 to t_8 can be similarly collapsed into transitions T_1 and T_2. The transitions T_1 and T_2 have therefore four colours. The colours can be taken as S_1J_1, S_2J_1, S_1J_2, and S_2J_2. The colour S_1J_1 represents, for example, the transition t_1, since it is enabled when S_1 and J_1 are available. The manufacturing system of Section 4.1 can therefore be represented by the coloured Petri net shown in Figure 8.

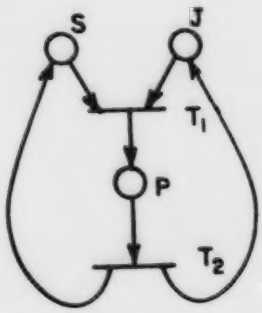


Figure 8. Coloured Petri net model of the FMS of Section 4.1.

The input and output functions of T_1 and T_2 can be obtained from the input and output functions of the transitions comprising these transitions. Consider, for example, the transition T_1. This can fire with respect to each one of its colours: S_1J_1, S_2J_1, S_1J_2 and S_2J_2. Therefore the input function from S to T_1 can be represented as

$$I(S,T_1) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \tag{34}$$

The matrix in Eq. (34) has two rows and four columns corresponding to the two colours of S and four colours of T_1. The first row contains 1 in the first and third place, since S_1 appears in the first and the third colour of transition T_1. The second row has nonzero elements in the second and fourth place, since S_2 appears in the second and the fourth colour of T_1. Similarly $I(J,T_1)$ is given by

$$I(J,T_1) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{35}$$

To find $O(P,T_1)$, notice that T_1 and P have the same colours. Therefore, $O(P,T_1) = I_4$, where I_4 is a (4×4) unit matrix. For the transition T_2 it is easy to see that $I(P,T_2) = I_4$, $O(S,T_2) = I(S,T_1)$, and $O(J,T_2) = I(J,T_1)$. Using these values, the incidence matrix C in Eq. (23) can be written in the following compact form

$$C = \begin{array}{c} \\ S \\ J \\ P \end{array} \begin{array}{c} \begin{array}{cc} T_1 & T_2 \end{array} \\ \begin{bmatrix} -I(S,T_1) & I(S,T_1) \\ -I(J,T_1) & I(J,T_1) \\ I_4 & -I_4 \end{bmatrix} \end{array} \tag{36}$$

We now give a formal definition of marked coloured Petri net.

Definition: A marked coloured Petri net is a 6-tuple (P, T, C, I, O, M_o) where

(i) $P = (p_1, p_2, \ldots, p_n)$ is a finite set of places.

(ii) $T = (t_1, t_2, \ldots, t_m)$ is a finite set of transitions.

(iii) $P \cup T \neq \phi$ and $P \cap T = \phi$.

(iv) C(p) and C(t) are sets of colours for each place $p \in P$ and each transition $t \in T$.

(v) I(p,t) and O(p,t) are, respectively, the input and output functions that assign a nonnegative integer to each pair of colours, $c' \in C(p)$ and $c'' \in C(t)$, for all $(p,t) \in P \times T$.

(vi) M_o is an initial marking that associates a nonnegative integer to each colour $c \in C(p)$, for all $p \in P$.

As for the Petri net, the coloured Petri net can also be represented by a directed bipartite graph. The places are represented by circles and transitions by bars. A directed arc is drawn from a place p_i to a transition t_j if $I(p_i,t_j) \neq 0$. Similarly, if $O(p_i,t_j) \neq 0$, a directed arc is drawn from t_j to p_i.

A transition t_j is enabled with respect to a colour $c(t_j)$ if the number of tokens in the input places of t_j is greater than or equal to the value of $I(p_i,t_j)$ corresponding to the colour $c(t_j)$ for all input places p_i. An enabled transition can only fire. When a transition t_j fires with respect to a colour $c(t_j)$, it removes $I(p_i,t_j)c(t_j)$ tokens from each of its input places and adds $O(p_i,t_j)c(t_j)$ tokens to each of its output places. It should be noted that a transition t may fire concurrently with respect to several of its colours. The incidence matrix C is a block matrix such that its ijth element $c_{ij} = O(p_i,t_j) - I(p_i,t_j)$ is a matrix of dimension $|C(p_i)| \times |C(t_j)|$ where $|S|$ means the cardinality of the set S. A block matrix $X = [x_1^T \; x_2^T \; \ldots \; x_n^T]^T$ will be called a p-invariant if

$$X^T C = 0$$

where x_i, i = 1,2 ...,n, is a matrix of dimension $|C(p_i)| \times q$. The value of q depends on the problem. For example, the p-invariants of the matrix C in Eq. (36) is given by $X = [x_1^T\ x_2^T\ x_1^T I(S,T_1) + x_2^T I(J,T_1)]^T$ where x_i's are (2 × 2) matrices. As there is a freedom in the choice of the matrices x_1 and x_2 there are two linearly independent p-invariants, namely $X_1 = [I_2\ 0\ I(S,T_1)]^T$ and $X_2 = (0\ I_2\ I(J,T_1))^T$. Here, I_2 is the unit matrix of dimension (2 × 2). These invariants can be used to show that the system is deadlock free.

Other concepts can be generalized in the same way (Jensen 1981; Narahari et al. 1985b). It should be noted that the definition of coloured Petri net reduces to the ordinary Petri net, since there is only one colour associated with each place and each transition.

As a further illustration of the use of coloured Petri net consider the example described in Section 4.2. By combining the places and transitions having similar interpretations in Figure 6 we obtain the coloured Petri net shown in Figure 9.

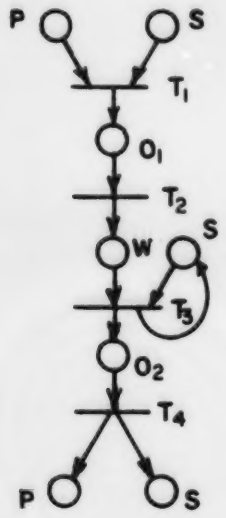


Figure 9. Coloured Petri net model of the system of Section 4.2.

The place P represents the places P_1 and P_2, and S represents S_1 and S_2, and so on. All the places and transitions in Figure 9 have two colours. The colour sets of P and S are (P_1,P_2) and (S_1,S_2), respectively. The colour sets for the rest of the places and the transitions T_1 - T_4 can be taken as (P_1,P_2). With this colour scheme, the incidence matrix is given by

$$C = \begin{array}{c} \\ P \\ S \\ O_1 \\ W \\ O_2 \end{array} \begin{array}{c} \begin{array}{cccc} T_1 & T_2 & T_3 & T_4 \end{array} \\ \begin{bmatrix} -I_2 & 0 & 0 & I_2 \\ -I_2 & 0 & I_2 - I(S,T_3) & I(S,T_3) \\ I_2 & -I_2 & 0 & 0 \\ 0 & I_2 & -I_2 & 0 \\ 0 & 0 & I_2 & -I_2 \end{bmatrix} \end{array}$$

where $I(S,T_3) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. The two linearly independent p-invariants can be readily calculated and are given by

$$X_1 = [I_2\ 0\ I_2\ I_2\ I_2]^T \tag{37}$$

$$X_2 = [0\ I_2\ I_2\ I_2\ I(S,T_3)]^T \tag{38}$$

Let the initial marking M_o be given as before. Then using the invariants (Eqs. (37) and (38)), we obtain the following relations for any marking M reachable from M_o.

$$M_o(P) = M(P) + M(O_1) + M(W) + M(O_2) \tag{39}$$

$$M_o(S) = M(S) + M(O_1) + M(W) + I(S,T_3)M(O_2) \tag{40}$$

Comparing Eqs. (39) and (40) to Eqs. (24) - (27) we notice that Eq. (39) corresponds to Eqs. (26) and (27) while Eq. (40) corresponds to Eqs. (24) and (25). For the marking M where $M(W) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, no transition is enabled and hence the system is deadlocked.

6.0 CONCLUSIONS

We have presented a comprehensive review of the Petri net and coloured Petri net theory and have shown how Petri nets and coloured Petri nets can be utilized for the description of flexible manufacturing systems. The Petri net invariants play an important role in analyzing properties (such as safeness, absence or presence of deadlock) of the modeled system. For small nets the invariants can be found easily. However, for large nets their determination is difficult. To alleviate this difficulty we have discussed a method that determines the invariants of the union of a finite number of nets from the known invariants of the individual nets, provided the individual nets have no transitions in common.

Petri nets and coloured Petri nets cannot be directly used for simulation and performance evaluation studies since the timing characteristics are not modeled by these nets. However, by associating times (deterministically or randomly) such studies can be conducted (Dubois et al. 1983; Pagnoni 1987).

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An Introduction to the Modeling and Analysis of Flexible Manufacturing Systems With Petri Nets

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SUMMARY/SOMMAIRE 15

This report presents a comprehensive review of the Petri nets and coloured Petri nets and describes their use in the modeling and analysis of flexible manufacturing systems.

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